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# Stopping power of charged particles due to ion wave excitations

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Stopping power due to ion wave excitations is derived for a charged particle moving in a two-component plasma. Unlike previous theories based on ion-acoustic-wave approximation (IAWA), the excitation of short-wavelength ion waves is taken into account. The obtained stopping power has a magnitude larger than that of IAWA. Stopping power at subsonic velocities, where stopping power in IAWA disappears, is even larger than that of supersonic velocities.

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Recently, the stopping power of plasmas has been intensively studied in connection with the inertial confinement fusion [1–4], the electron cooling of ion beams [5], and the drag force acting on dust particulates [6,7]. In general, the stopping power is composed of two types of energy loss processes: binary collisions and collective excitations [8]. In a two-component plasma, the excitation of ion waves may contribute to the stopping power as the latter in the velocity range $v_i \ll v \ll v_e$, where $v_i$ and $v_e$ are the thermal velocity of the ions and electrons, respectively. Stopping power due to the excitation of the ion acoustic wave has been obtained by Peter and Meyer-ter-Vehn and was found to be negligible in comparison with that of the single scattering processes [5]. It should be noted, however, that the ion-acoustic-wave approximation (IAWA) represents only a part of the ion wave mode. Indeed, as we shall show in this paper, the stopping power becomes one order of magnitude larger than that of IAWA if we take into account the whole range of the ion wave mode. It should be also noted that the ion acoustic wave may be excited only by particles at supersonic velocities while the ion wave may be excited also at subsonic velocities. As we shall show later, the stopping power in the subsonic region becomes larger than that of the supersonic region.

We calculate the stopping power using the linear response theory. The dielectric function for the ion wave is given by [9]

$$\epsilon(\mathbf{k},\omega)=1+\frac{k_e^2}{k^2}-\frac{\omega_{pi}^2}{\omega^2}+i\epsilon^i, \tag{1}$$

where $\omega_{pi}$ is the plasma frequency of the ions and $k_e$ is the Debye wave number of the electrons. The imaginary part $\epsilon^i$ represents the effect of damping, which may be assumed to be small as far as $k$ is smaller than the Debye wave number of ions, $k_i$. By solving $\epsilon(\mathbf{k},\omega)=0$, we obtain the dispersion relation for the ion wave

$$\omega_k=\frac{c_s k}{\sqrt{1+k^2/k_e^2}}, \tag{2}$$

where $c_s$ is the speed of ion acoustic waves. In the long-wavelength region, i.e., $k \ll k_e$, Eq. (2) is reduced to

$$\omega_k=c_s k, \tag{3}$$

which represents the ion acoustic mode.

The stopping power is defined as the energy loss per unit length, $S=-\mathbf{F}\cdot\mathbf{v}/v$, where the charged particle is assumed to move along the $z$ axis. The force acting on the particle is given by $\mathbf{F}=-\nabla\Phi[\mathbf{r}(t),t]$, where

$$\Phi(\mathbf{r},t)=\int\frac{d^3\mathbf{k}d\omega}{(2\pi)^4}\frac{4\pi}{k^2\epsilon(\mathbf{k},\omega)}\rho_{ext}(\mathbf{k},\omega)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} \tag{4}$$

is the effective potential [9,10]. The density of an external point charge is given by $\rho_{ext}(\mathbf{r},t)=q\delta(\mathbf{r}-\mathbf{v}t)$, and its Fourier transform by

$$\rho_{ext}(\mathbf{k},\omega)=2\pi q\delta(\omega-\mathbf{k}\cdot\mathbf{v}). \tag{5}$$

Using Eqs. (4) and (5), we obtain

$$S=\frac{4\pi q^2}{v}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{i\mathbf{k}\cdot\mathbf{v}}{k^2}\left[\frac{1}{\epsilon(\mathbf{k},\omega)}-1\right]. \tag{6}$$

The stopping power for the ion wave excitations is obtained by substituting Eq. (1) into (6):

$$S_{iw}=\frac{q^2}{v}\int_0^{k_c}dk\frac{\omega_k}{1+k_e^2/k^2}\int_{-1}^{1}d(\cos\theta)\cos\theta \times\left[\delta\left(\cos\theta-\frac{\omega_k}{kv}\right)-\delta\left(\cos\theta+\frac{\omega_k}{kv}\right)\right], \tag{7}$$

where $\mathbf{k}\cdot\mathbf{v}=kv\cos\theta$. The cut wave number $k_c$ may be taken as $k_c=k_i$ because at $k>k_i$ the ion wave mode is suppressed due to the Landau damping [9].

By observing the arguments of the $\delta$ function in Eq. (7), we find that $k$ must satisfy

$$k_e\sqrt{1/M^2-1}<k \tag{8}$$

 

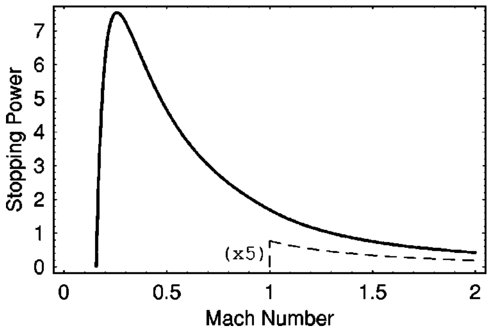


FIG. 1. Stopping power normalized by $q^2k_e^2$. The solid line represents $S_{iw}$. The dashed line shows $S_{IAWA}$ multiplied by the factor 5.

at subsonic velocities, where $M = v/c_s$ is the Mach number. Taking into account the condition (8), we obtain for $(1+k_i^2/k_e^2)^{-1/2} < M < 1$,

$$S_{iw} = \frac{q^2k_e^2}{M^2}\left[\ln\left(M\sqrt{1+\frac{k_i^2}{k_e^2}}\right)+\frac{1}{6}\left(\frac{1}{1+k_i^2/k_e^2}-M^6\right)\right] \tag{9}$$

and for $M>1$,

$$S_{iw} = \frac{q^2k_e^2}{M^2}\left[\ln\left(\sqrt{1+\frac{k_i^2}{k_e^2}}\right)+\frac{1}{6}\left(\frac{1}{1+k_i^2/k_e^2}-1\right)\right]. \tag{10}$$

The stopping power in IAWA is obtained by substituting Eq. (3) for $\omega_k$ in Eq. (7). In this case, we obtain

$$S_{IAWA} = \frac{q^2k_e^2}{2M^2}(1-\ln 2), \tag{11}$$

which is essentially identical to the result of Ref. [5]. In deriving Eq. (11) we have taken $k_c = k_e$ because IAWA holds only under the condition $k \ll k_e$. It should be noted that $S_{IAWA}$ does not depend on the temperature. In contrast, $S_{iw}$ becomes larger as $T_e/T_i$ $(=k_i^2/k_e^2)$ increases [11]. In the limit of $T_i \to 0$, $S_{iw}$ diverges in a logarithmic manner.

In Fig. 1 we show the velocity dependence of the stopping power divided by $q^2k_e^2$ at $T_e = 2$ eV and $T_i = 0.05$ eV. For comparison, the numerical value of $S_{IAWA}$ is shown. $S_{IAWA}$ has magnitude smaller than $S_{iw}$. (Note that the shown value is multiplied by the factor 5.) Since the ion acoustic waves can be excited only at the supersonic velocities, we have $S_{IAWA} = 0$ for $M < 1$.

The reason of the large difference between $S_{iw}$ and $S_{IAWA}$ is most clearly explained if we use the dispersion curves, as shown in Fig. 2. The $\delta$ functions in Eq. (7) represent the condition that both Eq. (2) and the well-known wave-particle resonance condition [12]

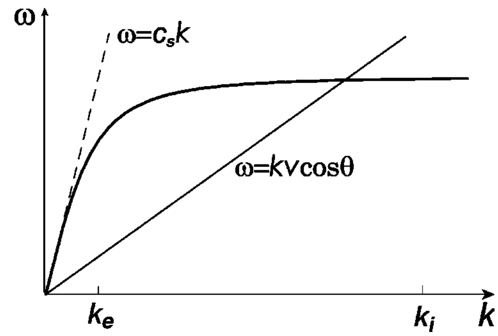


FIG. 2. The dispersion curve of the ion wave (solid curve). The dotted line represents IAWA. The solid line shows $\omega = \mathbf{k}\cdot\mathbf{v}$, which is the condition that the ion wave and the charged particle are in resonance. For fixed values of $\theta$ and $v, \omega$ and $k$ for the ion wave that can be excited are given at the point of intersection of the two solid lines.

$$\omega = kv\cos\theta \tag{12}$$

are satisfied. If we use IAWA, since Eq. (2) is reduced to $\omega_k = c_sk$, we obtain "the Cherenkov condition" $M = 1/\cos\theta$, which indicates that the ion acoustic waves are excited only by the supersonic particles. On the other hand, the lines given by Eqs. (2) and (12) always have a point of intersection even at the subsonic velocities. We note that the ion waves satisfying the condition $k_e < k$ contribute substantially to the stopping power.

Now we compare $S_{iw}$ with the stopping power due to binary collisions with the ions. Under the condition $v_i \ll v \ll v_e$, we may neglect the energy transfer to the electrons. The stopping power due to the binary collision is given by [13,14]

$$S_{ci} = \frac{4\pi(qe)^2}{m_iv^2}n\ln\left[\frac{2(a^2+b^2/4)^{1/2}}{b}\right], \tag{13}$$

where $a$ is of the order of the screening radius and $b = 2q/(m_0v^2), m_0$ being the reduced mass. Taking $a \approx \lambda_D, T_e = 2$ eV and $T_i = 0.05$ eV, we obtain $S_{iw}/S_{ci} \approx 0.16$ for $M = 1$. Though it is not so large, this value is not negligible for quantitative discussions.

Finally, let us point out that $S_{iw}$ represents the excitation rate of the wake potential [10] per unit length. Lemons *et al.* have shown that the wake potential may be not only formed at the supersonic velocities but also the subsonic ones if the short-wavelength contribution of the ion wave excitations is taken into account [15]. Also, they have shown that the subsonic wake potential becomes even larger. Their results coincide with ours.

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